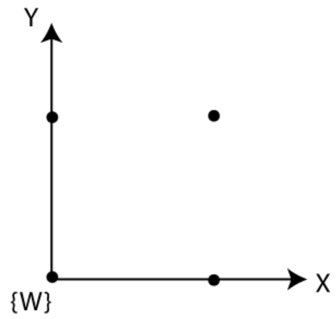
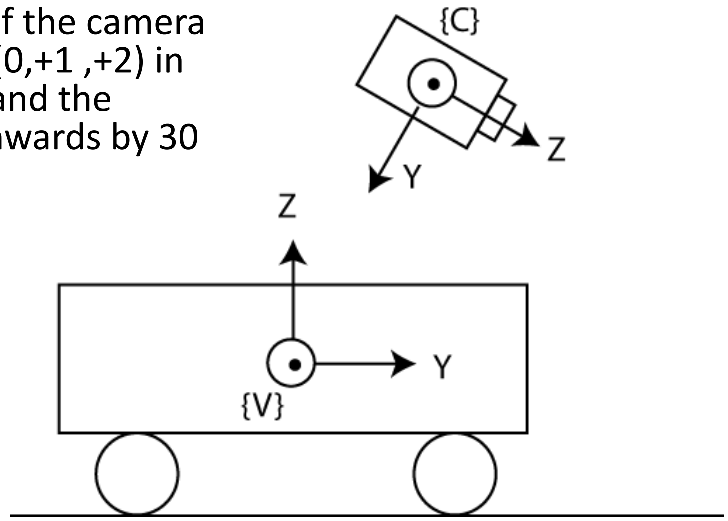
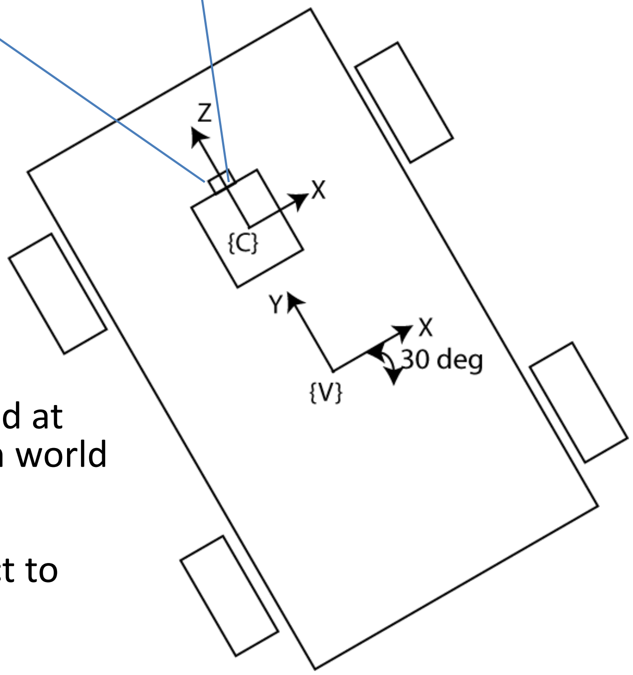


- A camera is mounted on a robot vehicle. The center of the camera is located at (X,Y,Z) = (0,+1 ,+2) in vehicle coordinates, and the camera is tilted downwards by 30 degrees





- The vehicle is located at (X,Y,Z) = (+4,-4,+1) in world coordinates, with a rotation angle of 30 degrees with respect to the world.

- The camera observes four points on the ground, with world coordinates at (0,0,0), (1,0,0), (1,1,0), and (0,1,0).

- Problem: transform the points from world coordinates to camera coordinates

Solution:
We need to convert the four points from world coordinates to camera coordinates. If we knew the transformation from the vehicle to the world (${}^{W}_{V}\mathbf{H}$) and the camera to the vehicle (${}^{V}_{C}\mathbf{H}$), we can combine the transformations using ${}^{C}\mathbf{P} = {}^{C}_{V}\mathbf{H}\ {}^{V}_{W}\mathbf{H}\ {}^{W}\mathbf{P}$, where ${}^{V}_{W}\mathbf{H} = \left({}^{W}_{V}\mathbf{H}\right)^{-1}$, etc.

The transformation from vehicle to world: The vehicle is aligned with the world except for a rotation of +30 degrees about the Z axis. So

$$ {}^{W}_{V}\mathbf{R} = \mathbf{R}_Z(30) = \begin{pmatrix} 0.8660 & -0.5000 & 0 \\ 0.5000 & 0.8660 & 0 \\ 0 & 0 & 1 \end{pmatrix}. $$

Note that a good check for whether the rotation matrix is correct is to look at the columns of the rotation matrix. The columns are the unit vectors of the vehicle coordinate system, expressed in the world coordinate system. So for example, ${}^{W}\hat{\mathbf{x}}_V = \begin{pmatrix} 0.8660 \\ 0.5000 \\ 0 \end{pmatrix}$ is the first column of R, and it says that the x axis of the vehicle points mostly in the +X direction of the world, somewhat in the +Y direction of the world, and not at all in the Z direction. This is true by looking at the diagram of the scene.

The center of the vehicle is located at (X,Y,Z) = (+4,-4,+1) in world coordinates, so ${}^{W}\mathbf{t}_{Vorg} = (4,-4,1)^T$. So

$$ {}^{W}_{V}\mathbf{H} = \begin{pmatrix} \mathbf{R}_Z(30) & {}^{W}\mathbf{t}_{Vorg} \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} $$

The transformation from camera to vehicle: The camera is aligned with the vehicle except for a rotation about the X axis. We rotate -90 degrees in order to flip the Y axis so that it points down instead of forward as in the vehicle frame. Then we need to rotate an additional -30 degrees to account for the camera tilting down. So

$$ {}^{V}_{C}\mathbf{H} = \begin{pmatrix} \mathbf{R}_X(-120) & {}^{V}\mathbf{t}_{Corg} \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} $$

FIND $${}^{W}_{V}H = \left[\begin{array}{c|c} {}^{W}_{V}R & {}^{W}\vec{t}_{VORG} \\ \hline 000 & 1 \end{array}\right]$$

$${}^{W}_{V}R = R_z(30^\circ) = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} {}^{W}\hat{x}_V & {}^{W}\hat{y}_V & {}^{W}\hat{z}_V \\ & & \end{bmatrix}$$

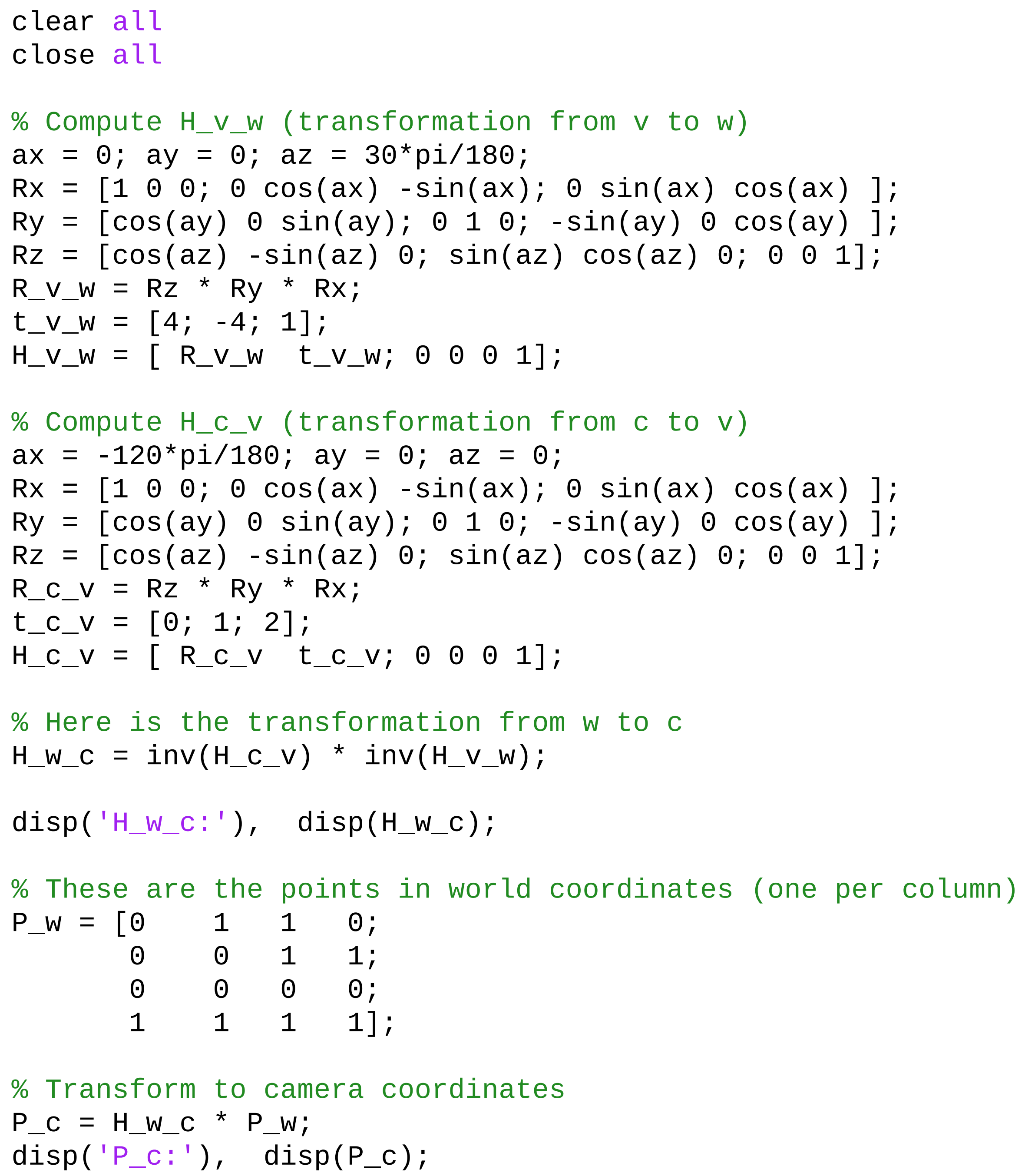
$${}^{W}_{V}H = \begin{bmatrix} .866 & -.5 & 0 & 4 \\ .5 & .866 & 0 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & \phi \end{bmatrix}$$

$${}^{v}_{c}R = R_x(-120^\circ)$$

$$= \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -.5 & +.866 \\ 0 & -.866 & -.5 \end{bmatrix} \\ \begin{array}{ccc} \uparrow & \uparrow & \\ {}^{v}\hat{x}_c & {}^{v}\hat{y}_c & \end{array} \end{array}$$

$${}^{v}_{c}H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -.5 & .866 & 1 \\ 0 & -.866 & -.5 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow {}^{c}_{v}H = \left({}^{v}_{c}H\right)^{-1}$$

$${}^{c}_{w}H = {}^{c}_{v}H \; {}^{v}_{w}H$$



```
clear all
close all

% Compute H_v_w (transformation from v to w)
ax = 0; ay = 0; az = 30*pi/180;
Rx = [1 0 0; 0 cos(ax) -sin(ax); 0 sin(ax) cos(ax) ];
Ry = [cos(ay) 0 sin(ay); 0 1 0; -sin(ay) 0 cos(ay) ];
Rz = [cos(az) -sin(az) 0; sin(az) cos(az) 0; 0 0 1];
R_v_w = Rz * Ry * Rx;
t_v_w = [4; -4; 1];
H_v_w = [ R_v_w  t_v_w; 0 0 0 1];

% Compute H_c_v (transformation from c to v)
ax = -120*pi/180; ay = 0; az = 0;
Rx = [1 0 0; 0 cos(ax) -sin(ax); 0 sin(ax) cos(ax) ];
Ry = [cos(ay) 0 sin(ay); 0 1 0; -sin(ay) 0 cos(ay) ];
Rz = [cos(az) -sin(az) 0; sin(az) cos(az) 0; 0 0 1];
R_c_v = Rz * Ry * Rx;
t_c_v = [0; 1; 2];
H_c_v = [ R_c_v  t_c_v; 0 0 0 1];

% Here is the transformation from w to c
H_w_c = inv(H_c_v) * inv(H_v_w);

disp('H_w_c:'),  disp(H_w_c);

% These are the points in world coordinates (one per column)
P_w = [0    1   1   0;
       0    0   1   1;
       0    0   0   0;
       1    1   1   1];

% Transform to camera coordinates
P_c = H_w_c * P_w;
disp('P_c:'),  disp(P_c);
```

- Results:

```
H_w_c:
    0.8660    0.5000         0   -1.4641
    0.2500   -0.4330   -0.8660    0.3660
   -0.4330    0.7500   -0.5000    5.3660
         0         0         0    1.0000
P_c:
   -1.4641   -0.5981   -0.0981   -0.9641
    0.3660    0.6160    0.1830   -0.0670
    5.3660    4.9330    5.6830    6.1160
    1.0000    1.0000    1.0000    1.0000
```